# carlomat_3.1 — A TOOL FOR DESCRIBING PHOTON RADIATION IN ELECTRON–POSITRON ANNIHILATION TO HADRONS AT LOW ENERGIES* **

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A current version of the multipurpose program carlomat offers a possibility of taking into account either the initial- or final-state radiation separately, or both at a time. It allows to include the electromagnetic charged pion form factor in processes with charged pion pairs and to perform the U(1) gauge invariance tests in an easy way. In this paper, I will illustrate how those new capabilities of the program can be utilized in the description of the electron–positron annihilation to hadrons at low energies.

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## 1. Motivation

The hadronic contribution to vacuum polarization can be derived, with the help of dispersion relations, from the energy dependence of the ratio

$$R_\gamma(s) \equiv \sigma^{(0)}\left(e^+e^- \to \gamma^* \to \text{hadrons}\right) \Big/ \frac{4\pi\alpha^2}{3s}\,. \tag{1}$$

One of the main issues is $R_\gamma(s)$ in the region from 1.2 to 2.0 GeV, where more than 30 exclusive channels must be measured. To obtain reliable theoretical predictions for that many hadronic processes is a challenge indeed.

It is obvious that the correct description of the most relevant hadronic channels as, *e.g.*, $\pi^+\pi^-$, requires the inclusion of radiative corrections. This demand is met, *e.g.*, by the dedicated Monte Carlo (MC) generator

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PHOKHARA [1]. However, it is probably enough to have the leading order (LO) predictions for many sub-dominant channels, with three or more particles in the final state. If those channels are measured with the method of radiative return, as is done by KLOE, BaBar and BES, the predictions must also include radiation of photons, both from the initial (ISR) and final (FSR) state.

Production of hadrons at low energies, as well as the photon radiation off them, is usually described in the framework of some effective model which often includes quite a number of interaction vertices and mixing terms. This means that the number of Feynman diagrams of such multiparticle processes may become quite big. Therefore, there is a strong need for full automation of the MC code generation.

A promising theoretical framework for the description of $e^+e^- \to$ hadrons at low energies is the Hidden Local Symmetry (HLS) model. The HLS model, supplemented by isospin and SU(3) breaking effects, works surprisingly well up to 1.05 GeV, just including the $\phi$ meson [2, 3].

The MC programs for description of processes $e^+e^- \to$ hadrons at low centre-of-mass energies can be generated automatically with program carlomat_3.0 [4]. The program, among many other useful properties, incorporates a photon–vector meson mixing, includes the Feynman interaction vertices of HLS model and the effective Lagrangian of the electromagnetic (EM) interaction of nucleons, and introduces several new options to enable a better control over the effective models implemented.

A current version of the program, carlomat_3.1 [5], offers a possibility of taking into account either the ISR or FSR separately, or both at a time, it allows to include the EM charged pion form factor in processes with charged pion pairs and to perform the U(1) gauge invariance tests in an easy way. In this paper, I will illustrate how those new capabilities of the program can be utilized in the description of the electron–positron annihilation to hadrons at low energies.

## 2. Utilizing carlomat_3.1

The theoretical model used here was described in detail in [6], where the Feynman rules of the HLS model are depicted in Figs. 3 and 7–9. This time, however, we neglect the couplings of $e^+e^-$ and $\mu^+\mu^-$ to both $Z^0$ and Higgs bosons, and not only to the latter as was done in [6], which actually means that the Standard Model (SM) part of the model implemented in the program is reduced to QED. Even within this relatively simple model, the number of Feynman diagrams of the radiative processes

$$e^+e^- \to \pi^+\pi^-\mu^+\mu^-\gamma\,, \tag{2}$$

$$e^+e^- \rightarrow \pi^+\pi^-e^+e^-\gamma\,, \tag{3}$$
$$e^+e^- \rightarrow \pi^+\pi^-\pi^+\pi^-\gamma \tag{4}$$

is quite big, amounting to 451, 834 and 4174 for processes (2), (3) and (4), respectively. Those numbers can be substantially reduced if we combine the vector meson mixing terms and vertices of vector meson decays into the charged pion pair into an effective photon-charged pion interaction vertex which is defined in terms of the electromagnetic charged pion form factor $F_\pi(q^2)$ according to Eqs. (3) and (9) of [6]. Thus, just keeping the couplings: $\gamma\pi^+\pi^-$, $\pi^0\gamma\gamma$, $\pi^0\gamma\rho^0$, $\gamma\gamma\pi^+\pi^-$, $\gamma\pi^0\pi^+\pi^-$ and the single mixing term $\gamma\rho^0$, we get 75, 150 and 200 Feynman diagrams of processes (2), (3) and (4), respectively. How this modification of the model changes theoretical predictions for a considered process is illustrated in Fig. 1, where the differential cross sections of process (2) at $\sqrt{s} = 1$ and 1.5 GeV, computed with the following cuts:

$$E_\gamma > 0.01 \text{ GeV}\,, \qquad 5^\circ < \theta_{\gamma\mathrm{b}} < 175^\circ \tag{5}$$

on the photon energy $E_\gamma$ and photon angle with respect to the beam $\theta_{\gamma\mathrm{b}}$, are plotted as functions of the invariant mass of the $\pi^+\pi^-\mu^+\mu^-$-system. In both panels, the shaded histogram shows the prediction of the model with the pion form factor $F_\pi(q^2)$ and the dashed line shows the prediction of the model with 451 Feynman diagrams and fixed couplings. As can be seen from Fig. 1, the difference is not big for this particular process, but for other processes it may become quite substantial.

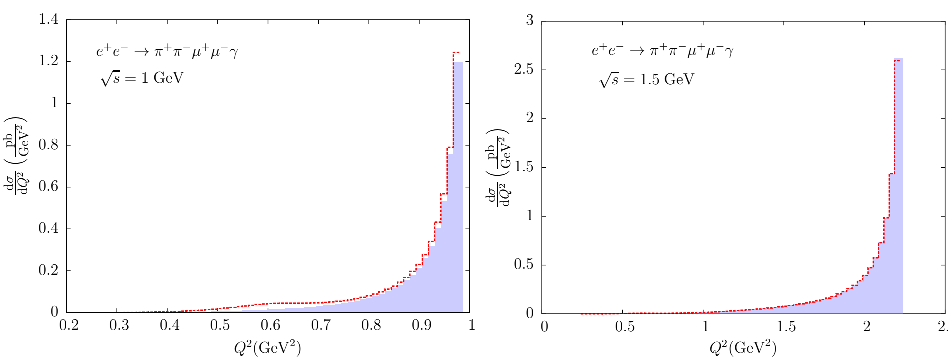


Fig. 1. Differential cross sections of (2) at $\sqrt{s} = 1$ and 1.5 GeV, with cuts given by (5), as functions of the invariant mass of the $\pi^+\pi^-\mu^+\mu^-$-system.

Although carlomat_3.1 does not generate automatically kinematics that would flatten singular behaviour of the $t$-channel Feynman diagrams, the program can be used to obtain predictions for processes that involve such diagrams, as *e.g.* process (3), provided that the contribution of the $t$-channel diagrams to the cross section is reduced by imposing angular cuts on the final-state particles. This is illustrated in Fig. 2, where the differential cross sections of process (3) at $\sqrt{s} = 0.8$ and 1 GeV are plotted as functions of the invariant mass of the $\pi^+\pi^-e^+e^-$-system. The cross sections have been computed within the model with fixed HLS couplings, as discussed in the previous paragraph, and the following cuts have been imposed on the angles between lepton–beam $\theta_{l\mathrm{b}}$, photon–beam $\theta_{\gamma\mathrm{b}}$ and photon–lepton $\theta_{\gamma l}$, and on the photon energy $E_\gamma$:

$$20^\circ < \theta_{l\mathrm{b}}\,,\, \theta_{\gamma\mathrm{b}} < 160^\circ, \qquad 10^\circ < \theta_{\gamma l} < 170^\circ, \qquad E_\gamma > 0.01\ \mathrm{GeV}. \tag{6}$$

The shaded histograms in both panels of Fig. 2 show the ISR, and the solid lines represent the full LO cross sections. Thus, the difference between the two illustrates the FSR effects. The small peaks at about 3.5 and 4 GeV$^2$ are due to statistical fluctuations, which arise as the multi-channel integration routine generated automatically by carlomat is not perfect. It covers basically only the $s$-channel poles.

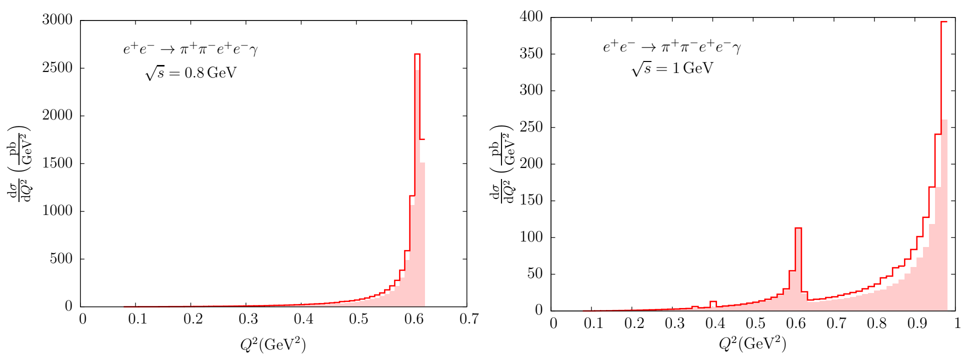


Fig. 2. Differential cross sections of (3) at $\sqrt{s} = 0.8$ and 1 GeV, with cuts given by (6), as functions of the invariant mass of the $\pi^+\pi^-e^+e^-$-system.

Another useful option offered by carlomat_3.1 is a possibility of performing tests of the U(1) gauge invariance in an easy way, just by setting a value of flag `igauge=1,2,...` in `carlocom.f`, a main routine of the MC computation part of the program, corresponding to the photon polarization four vector which is going to be replaced with its four momentum. If the test is well-satisfied, then the cross section should drop by about 30 orders of magnitude with respect to the computation with `igauge=0`, when no gauge invariance test is performed. However, if the Feynman rules implemented in the program are incomplete, or the four-momentum transfer in a subset of interaction vertices is set inconsistently, which may happen in the process of automatic generation of amplitudes, then the drop in the value of cross section will not certainly be so spectacular any more, amounting to just a few orders of magnitude in some situations.

## 3. Summary and outlook

A multipurpose program carlomat allows to generate codes for MC calculations of the leading order cross sections and simulations of, among others, multiparticle processes of $e^+e^- \to$ hadrons at low centre-of-mass energies in the framework of HLS model. A current version of the program offers a possibility of taking into account either the ISR or FSR separately, or both at a time, which is a very useful option if the cross sections of $e^+e^- \to$ hadrons are determined with the radiative return method. Moreover, it allows to include the EM charged pion form factor in processes with charged pion pairs and to perform the U(1) gauge invariance tests in an easy way. As all couplings of the HLS model which have been implemented in the program may contain momentum dependence, one can, in principle, introduce some higher order effects through them, thus going beyond the leading order description of the considered processes.

However, it should be stressed here that the program in its present form is far from being complete. This means that there is no guarantee that one will obtain a satisfactory description of any processes of $e^+e^- \to$ hadrons at low centre-of-mass energies if one applies the program blindly, without spending some additional work on properly selecting many available program options and adjusting the couplings of the HLS model in a proper way in order to fit the data.

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